High School Mathematics in Middle School[4]

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade[5] or earlier so they can take college-level mathematics in high school.[6] Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Enrollment in both AP Statistics and AP Calculus, for example, have essentially doubled over the last decade (College Board, 2009). There is also powerful research showing that among academic factors, the strongest predictor of whether a student will earn a bachelor's degree is the highest level of mathematics taken in high school (Adelman, 1999). A recent study completed by The College Board confirms this. Using data from 65,000 students enrolled in 110 colleges, students' high school coursework was evaluated to determine which courses were closely associated with students' successful performance in college. The study confirmed the importance of a rigorous curriculum throughout a students' high school career. Among other conclusions, the study found that students who took more advanced courses, such as Pre-Calculus in the 11th grade or Calculus in 12th grade, were more successful in college. Students who took AP Calculus at any time during their high school careers were most successful (Wyatt & Wiley, 2010). And even as more students are enrolled in more demanding courses, it does not necessarily follow that there must be a corresponding decrease in engagement and success (Cooney & Bottoms, 2009, p. 2).

At the same time, there are cautionary tales of pushing underprepared students into the first course of high school mathematics in the eighth grade. The Brookings Institute's 2009 Brown Center Report on American Education found that the NAEP scores of students taking Algebra I in the eighth grade varied widely, with the bottom ten percent scoring far below grade level. And a report from the Southern Regional Education Board, which supports increasing the number of middle students taking Algebra I, found that among students in the lowest quartile on achievement tests, those enrolled in higher-level mathematics had a slightly higher failure rate than those enrolled in lower-level mathematics (Cooney & Bottoms, 2009, p. 2). In all other quartiles, students scoring similarly on achievement tests were less likely to fail if they were enrolled in more demanding courses. These two reports are reminders that, rather than skipping or rushing through content, students should have appropriate progressions of foundational content to maximize their likelihoods of success in high school mathematics.

It is also important to note that notions of what constitutes a course called "Algebra I" or "Mathematics I" vary widely. In the CCSS, students begin preparing for algebra in Kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to typical Algebra I courses—namely linear equations, inequalities, and functions—is found in the 8th grade CCSS. The Algebra I course described here ("High School Algebra I"), however, is the first formal algebra course in the Traditional Pathway (concepts from this Algebra I course are developed across the first two courses of the integrated pathway). Enrolling an eighth-grade student in a watered down version of either the Algebra I course or Mathematics I course described here may in fact do students a disservice, as mastery of algebra including attention to the Standards for Mathematical Practice is fundamental for success in further mathematics and on college entrance examinations. As mentioned above, skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later, because students may be denied the opportunity for a rigorous Algebra I or Mathematics I course and may miss important content from eighth-grade mathematics.

Middle School Acceleration

Taking the above considerations into account, as well as the recognition that there are other methods for accomplishing these goals, the Achieve Pathways Group endorses the notion that all students who are ready for rigorous high school mathematics in eighth grade should take such courses (Algebra I or Mathematics I), and that all middle schools should offer this opportunity to their students. To prepare students for high school mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. The Achieve Pathways Group has developed two compacted course sequences, one designed for districts using a traditional Algebra I – Geometry – Algebra II high school sequence, and the other for districts using an integrated sequence, which is commonly found internationally. Both are based on the idea that content should compact 3 years of content into 2 years, at most. In other words, compacting content from 2 years into 1 year would be too challenging, and compacting 4 years of content into 3 years starting in grade 7 runs the risk of compacting across middle and high schools. As such, grades 7, 8, and 9 were compacted into grades 7 and 8 (a 3:2 compaction). As a result, some 8th grade content is in the 7th grade courses, and high school content is in 8th grade.

[4]This section refers to mathematics content, not high school credit. The determination for high school credit is presumed to be made by state and local education agencies.
[5]Either 8th Grade Algebra I or Accelerated Mathematics I.
[6]Such as Calculus or Advanced Statistics.

The compacted traditional sequence, or, "Accelerated Traditional," compacts grades 7, 8, and High School Algebra I into two years: "Accelerated 7th Grade" and "8th Grade Algebra I." Upon successfully completion of this pathway, students will be ready for Geometry in high school. The compacted integrated sequence, or, "Accelerated Integrated," compacts grades 7, 8, and Mathematics I into two years: "Accelerated 7th Grade" and "8th Grade Mathematics I." At the end of 8th grade, these students will be ready for Mathematics II in high school. While the K-7 CCSS effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Accelerated 7th Grade course to make the 8th Grade courses more manageable.

The Achieve Pathways Group has followed a set of guidelines[7] for the development of these compacted courses.

1. **Compacted courses should include the same Common Core State Standards as the non-compacted courses.** It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.

2. **Decisions to accelerate students into the Common Core State Standards for high school mathematics before ninth grade should not be rushed.** Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.

3. **Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning.** Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. **A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.** Traditionally, students taking high school mathematics in the eighth grade are expected to take Precalculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career (see Fourth Courses section of this paper for further detail).

Other Ways to Accelerate Students

Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra II, or Precalculus and Statistics).
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.
- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Mathematical Practices.[8]
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade.
- Creating a hybrid Algebra II-Precalculus course that allows students to go straight to Calculus.

A combination of these methods and our suggested compacted sequences would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career. The compacted sequences begin here:

[7]Based on work published by Washington Office of the Superintendent of Public Schools, 2008
[8]As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses likely be enormously fast.

Overview of the Accelerated Traditional Pathway for the Common Core State Mathematics Standards

This table shows the domains and clusters in each course in the Accelerated Traditional Pathway. The standards from each cluster included in that course are listed below each cluster. For each course, limits and focus for the clusters are shown in italics. For organizational purposes, clusters from 7th Grade and 8th Grade have been situated in the matrix within the high school domains.

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses*
Number and Quantity	The Real Number System	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. 7.NS.1a, 1b, 1c, 1d, 2a, 2b, 2c, 2d, 3 • Know that there are numbers that are not rational, and approximate them by rational numbers. 8.NS.1, 2 • Work with radicals and integer exponents. 8.EE.1, 2, 3, 4	• Extend the properties of exponents to rational exponents. N.RN.1, 2 • Use properties of rational and irrational numbers. N.RN.3.			
	Quantities	• Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1, 2a, 2b, 2c, 2d, 3	• Reason quantitatively and use units to solve problems. *Foundation for work with expressions, equations and functions* N.Q.1, 2, 3			

*The (+) standards in this column are those in the Common Core State Standards that are not included in any of the Accelerated Traditional Pathway courses. They would be used in additional courses developed to follow Algebra II.

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Number and Quantity	The Complex Number System				• Perform arithmetic operations with complex numbers. N.CN.1, 2 • Use complex numbers in polynomial identities and equations. *Polynomials with real coefficients* N.CN.7, (+) 8, (+) 9	• Perform arithmetic operations with complex numbers. (+) N.CN.3 • Represent complex numbers and their operations on the complex plane. (+) N.CN.4, 5, 6
	Vector Quantities and Matrices					• Represent and model with vector quantities. (+) N.VM.1, 2, 3 • Perform operations on vectors. (+) N.VM.4a, 4b, 4c, 5a, 5b • Perform operations on matrices and use matrices in applications. (+) N.VM.6, 7, 8, 9, 10, 11, 12
Algebra	Seeing Structure in Expressions	• Use properties of operations to generate equivalent expressions. 7.EE.1, 2 • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.. 7.EE.3, 4a, 4b	• Interpret the structure of expressions. *Linear, exponential, quadratic* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. *Quadratic and exponential* A.SSE.3a, 3b, 3c		• Interpret the structure of expressions. *Polynomial and rational* A.SSE.1a, 1b, 2 • Write expressions in equivalent forms to solve problems. A.SSE.4	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Arithmetic with Polynomials and Rational Expressions		• Perform arithmetic operations on polynomials. *Linear and quadratic* A.APR.1		• Perform arithmetic operations on polynomials. *Beyond quadratic* A.APR.1 • Understand the relationship between zeros and factors of polynomials. A.APR.2, 3 • Use polynomial identities to solve problems. A.APR.4, (+) 5 • Rewrite rational expressions. *Linear and quadratic denominators* A.APR.6, (+) 7	
	Creating Equations		• Create equations that describe numbers or relationships. *Linear, quadratic, and exponential (integer inputs only) for A.CED.3, linear only* A.CED. 1, 2, 3, 4		• Create equations that describe numbers or relationships. *Equations using all available types of expressions, including simple root functions* A.CED.1, 2, 3, 4	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Algebra	Reasoning with Equations and Inequalities	• Understand the connections between proportional relationships, lines, and linear equations. 8.EE.5, 6 • Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.7a, 7b	• Understand solving equations as a process of reasoning and explain the reasoning. *Master linear, learn as general principle* A.REI.1 • Solve equations and inequalities in one variable. *Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions* A.REI.3, 4a, 4b • Analyze and solve linear equations and pairs of simultaneous linear equations. 8.EE.8a, 8b, 8c • Solve systems of equations. *Linear-linear and linear-quadratic* A.REI.5, 6, 7 • Represent and solve equations and inequalities graphically. *Linear and exponential; learn as general principle* A.REI.10, 11, 12		• Understand solving equations as a process of reasoning and explain the reasoning. *Simple radical and rational* A.REI.2 • Represent and solve equations and inequalities graphically. *Combine polynomial, rational, radical, absolute value, and exponential functions* A.REI.11	• Solve systems of equations. (+) A.REI.8, 9

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Functions	Interpreting Functions		• Define, evaluate, and compare functions. 8.F.1, 2, 3 • Understand the concept of a function and use function notation. *Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences* F.IF.1, 2, 3 • Use functions to model relationships between quantities. 8.F.4, 5 • Interpret functions that arise in applications in terms of a context. *Linear, exponential, and quadratic* F.IF.4, 5, 6 • Analyze functions using different representations. *Linear, exponential, quadratic, absolute value, step, piecewise-defined* F.IF.7a, 7b, 7e, 8a, 8b, 9		• Interpret functions that arise in applications in terms of a context. *Emphasize selection of appropriate models* F.IF.4, 5, 6 • Analyze functions using different representations. *Focus on using key features to guide selection of appropriate type of model function* F.IF.7b, 7c, 7e, 8, 9	• Analyze functions using different representations. *Logarithmic and trigonometric functions* (+) F.IF.7d

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Functions	Building Functions		• Build a function that models a relationship between two quantities. *For F.BF.1, 2, linear, exponential, and quadratic* F.BF.1a, 1b, 2 • Build new functions from existing functions. *Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only* F.BF.3, 4a		• Build a function that models a relationship between two quantities. *Include all types of functions studied* F.BF.1b • Build new functions from existing functions. *Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types* F.BF.3, 4a	• Build a function that models a relationship between two quantities. (+) F.BF.1c • Build new functions from existing functions. (+) F.BF.4b, 4c, 4d, 5
	Linear, Quadratic, and Exponential Models		• Construct and compare linear, quadratic, and exponential models and solve problems. F.LE.1a, 1b, 1c, 2, 3 • Interpret expressions for functions in terms of the situation they model. *Linear and exponential of form $f(x) = b^x + k$* F.LE.5		• Construct and compare linear, quadratic, and exponential models and solve problems. *Logarithms as solutions for exponentials* F.LE.4	
	Trigonometric Functions				• Extend the domain of trigonometric functions using the unit circle. F.TF.1, 2 • Model periodic phenomena with trigonometric functions. F.TF.5 • Prove and apply trigonometric identities. F.TF.8	• Extend the domain of trigonometric functions using the unit circle. (+) F.TF.3, 4 • Model periodic phenomena with trigonometric functions. (+) F.TF. 6, 7 • Prove and apply trigonometric identities. (+) F.TF. 9

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Congruence	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Focus on constructing triangles* 7.G.2 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.1a, 1b, 1c, 2, 5 • For 8.G.5, informal arguments to establish angle sum and exterior angle theorems for triangles and angles relationships when parallel lines are cut by a transversal		• Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 • Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof* G.CO.6, 7, 8 • Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11 • Make geometric constructions. *Formalize and explain processes* G.CO.12,13		
	Similarity, Right Triangles, and Trigonometry	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Scale drawings* 7.G.1 • Understand congruence and similarity using physical models, transparencies, or geometric software. 8.G.3, 4, 5 • For 8.G.5, informal arguments to establish the angle-angle criterion for similar triangles		• Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 • Prove theorems involving similarity. G.SRT.4, 5 • Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8 • Apply trigonometry to general triangles. G.SRT.9. 10, 11		

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Geometry	Circles			• Understand and apply theorems about circles. G.C.1, 2, 3, (+) 4 • Find arc lengths and areas of sectors of circles. *Radian introduced only as unit of measure* G.C.5		
	Expressing Geometric Properties with Equations			• Translate between the geometric description and the equation for a conic section. G.GPE.1, 2 • Use coordinates to prove simple geometric theorems algebraically. *Include distance formula; relate to Pythagorean theorem* G.GPE. 4, 5, 6, 7		• Translate between the geometric description and the equation for a conic section. (+) G.GPE.3
	Geometric Measurement and Dimension	• Draw, construct, and describe geometrical figures and describe the relationships between them. *Slicing 3-D figures* 7.G.3 • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G.4, 5, 6 • Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. 8.G.9	• Understand and apply the Pythagorean theorem. *Connect to radicals, rational exponents, and irrational numbers* 8.G.6, 7, 8	• Explain volume formulas and use them to solve problems. G.GMD.1, 3 • Visualize the relation between two-dimensional and three-dimensional objects. G.GMD.4		• Explain volume formulas and use them to solve problems. (+) G.GMD.2
	Modeling with Geometry			• Apply geometric concepts in modeling situations. G.MG.1, 2, 3		

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Statistics and Probability	Interpreting Categorical and Quantitative Data		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.1, 2, 3 • Investigate patterns of association in bivariate data. 8.SP.1, 2, 3, 4 • Summarize, represent, and interpret data on two categorical and quantitative variables. *Linear focus; discuss general principle* S.ID.5, 6a, 6b, 6c • Interpret linear models. S.ID.7, 8, 9		• Summarize, represent, and interpret data on a single count or measurement variable. S.ID.4	
	Making Inferences and Justifying Conclusions	• Use random sampling to draw inferences about a population. 7.SP.1, 2 • Draw informal comparative inferences about two populations. 7.SP.3, 4			• Understand and evaluate random processes underlying statistical experiments. S.IC.1, 2 • Make inferences and justify conclusions from sample surveys, experiments and observational studies. S.IC.3, 4, 5, 6	

	Domains	Accelerated 7th Grade	8th Grade Algebra I	Geometry	Algebra II	Fourth Courses
Statistics and Probability	Conditional Probability and the Rules of Probability	• Investigate chance processes and develop, use, and evaluate probability models. 7.SP.5, 6, 7a, 7b, 8a, 8b, 8c		• Understand independence and conditional probability and use them to interpret data. *Link to data from simulations or experiments* S.CP.1, 2, 3, 4, 5 • Use the rules of probability to compute probabilities of compound events in a uniform probability model. S.CP.6, 7, (+) 8, (+) 9		
	Using Probability to Make Decisions			• Use probability to evaluate outcomes of decisions. *Introductory; apply counting rules* (+) S.MD.6, 7	• Use probability to evaluate outcomes of decisions. *Include more complex situations* (+) S.MD.6, 7	• Calculate expected values and use them to solve problems. (+) S.MD.1, 2, 3, 4 • Use probability to evaluate outcomes of decisions. (+) S.MD. 5a, 5b